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ANALYZING TRANSPORTATION AND STORAGE SYSTEMS AS CAPACITATED NETWORKS*

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ABSTRACT

This paper discusses alternative methodologies for the economic analysis of transportation and storage systems with emphasis on developing countries. The capacitated network approach is presented as a robust, comprehensive and flexible methodology that may be useful for the analysis of certain problems. The methodology is described along with the Fulkerson OKA algorithm. An example is presented based on Brazilian research to demonstrate the methodology and sample results.

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Introduction

A major problem in the economic analysis of transportation and storage systems is the selection of the most robust, comprehensive and flexible methodology to test alternative strategies designed to improve efficiencies and reduce costs. The problem is particularly difficult when analyzing such issues in developing countries with limited capacities to solve computer algorithms.

Agricultural economists have frequently used various types of transportation models to analyze agricultural transportation and storage problems. Little use has been made of the capacitated network (CN) approach which, for certain problems, offers advantages of simplicity, flexibility, ease of use and interpretation, and less demands on computer time and capacity. The purpose of this paper is to briefly describe the CN approach, and provide an example based on an application to a Brazilian problem. The Fulkerson algorithm is used to solve the problem. References are provided for readers who want to pursue the approach and algorithm in greater detail.

A Comparison of Transportation and Network Models

Agricultural economists have traditionally used three types of models to study the allocation of commodities from surplus (producing) regions to deficit (consuming) regions: (1) the simple transportation models (ST); (2) the transshipment model (TS); and (3) the spatial price equilibrium (SPE) model. The assumptions of the simple transportation model are outlined below. The symbols in parentheses identify those which relax the respective assumption:

1. There are direct links between each origin and destination (CN, TS, SPE).
2. There are no capacity constraints on the links (CN, SPE).
3. There is no storage (CN, TS, SPE).
4. Unit transport costs are independent of the number of units shipped (CN, SPE).
5. No backhauls exist (CN, TS, SPE).
6. The amounts of "surplus" and "deficit" are known and fixed in each region (SPE).
7. The product shipped is homogeneous (CN, TS, SPE).
8. Perfect competition prevails.
9. Regions may be represented by points.

Due to its restrictive assumptions, the usefulness of the simple transportation model is limited to relatively simple problems such as finding the least-cost solution for shipping a commodity directly from a series of origins (e.g., factories) to a series of destinations (e.g., warehouses). The transshipment model allows one transshipment point between each origin-destination (O-D) pair. This additional flexibility permits analysis of more complex problems such as the determination of the optimum combination of processing, storage and interregional commodity movement patterns [King and Henry; Kriebel]. The model can also be used in optimal location analysis [King and Logan; Rhody; B. Wright; Goldman; Casetti; Ladd and Lifferth].

The spatial price equilibrium model is the only model to relax assumption six thereby permitting endogenous determination of equilibrium demand and supply. It is thus used to project trade flows where statistics do not permit direct mapping of interregional patterns of trade [Morrill and Garrison; King; Takayama and Judge; Walker]. As King and Logan (p. 99) point out, however, relaxing this assumption has a high cost in terms of problem formulation and computational efficiency. For a case with 30 regions, the SPE formulation requires 90 equations and 1,800 activities, while the equivalent transshipment formulation involves only a 60 x 60 matrix.

A quadratic programming formulation of the SPE model requires that the demand and supply schedules be known, continuous and linear. In LDC's, producing regions are often well defined with products flowing to a few major population centers or ports so that quantities and flows may be estimated exogeneously. In fact, endogenous estimates of supply obtained from linear supply functions may be no more accurate than exogenous estimates given output variations in any given year due to weather, area cultivated and other factors.

The ST and TS models are normally not used to model the physical characteristics of transport-storage systems.[1/] They are limited in this respect by the assumptions that all O-D

[1/] The failure to model these characteristics may not be an important limitation for studies in DC's if carriers are not saturated for any length of time and the physical infrastructure is generally not subject to rapid alteration.

linkages have infinite capacities, and that no more than one transshipment point may exist between an O-D pair (TS model). A general linear programming model can incorporate either maximum capacities on given linkages or multiple transshipment points, but the conceptual and computational complexity increases rapidly. In fact, inclusion of these real-world features of transport-storage systems can exhaust computer capabilities on very small problems [Ford and Fulkerson, p. 93]. Computational feasibility using linear programming may thus require the assumption that goods will be transported on the least cost carrier (or combination of carriers) in any quantity at a constant unit cost [Fedeler, Heady and Koo, p. 460].

In contrast, the capacitated network model explicitly incorporates upper and lower capacities on all linkages. The generality of the CN approach is seen by the number of assumptions which can be relaxed. Furthermore, the conceptual clarity of the capacitated network and the computational efficiency of its specialized algorithms make it the preferred alternative for solving a broad class of transportation problems. Applications found in the literature include studies of urban transportation [Gauthier; Muraco], coal shipment in the Great Lakes area [King, et al.] and fruit distribution in New Zealand [Sinclair and Kissling]. McCurdy et al. analyzed containerized shipping patterns on the South Island in an unpublished paper. Kane analyzed the economic impact of rail abandonment on the shipment of grains in Ohio. C. Wright and Feldens studied various features of the transportation and storage of grains in Southern Brazil.

The capacitated network approach is a promising tool for use by agricultural economists, particularly in the study of rapidly developing regions, since it enables the research to incorporate capacity limitations on linkages and to treat the following issues:

1) the efficiency of the entire transport-storage network;
2) the identification of existing bottlenecks and those which may appear with projected increases in agricultural output;
3) the costs and capacity characteristics of individual links in the network;
4) the quantitative effects of specific improvements in the network in terms of accessibility of nodes (centers) within the network and reduction in total shipping costs; and
5) the effect of nonlinear cost functions.

The Capacitated Network Model

The power and simplicity of the capacitated network approach in analyzing a transport-storage system can be appreciated by considering some illustrations. Figure 1 is a simplified representation of the transportation system for parts of the states of Sao Paulo and Parana in Brazil.[2/] Londrina (2) is the center

[2/] The examples cited reflect some general transport-storage problems of the area, but are used only for purposes of illustration. All figures cited are hypothetical.

of an established producing region and Cascavel (1) is a rapidly expanding frontier area. Both regions are experiencing dramatic agricultural development. The highway system has modern main arteries and is fairly complete, but the rail system is antiquated and serves only part of the area. Grains produced at (1) and (2) must meet domestic demands (estimated exogenously) in the state capitals Sao Paulo (5) and Curitiba (6). The remainder is exported through the ports of Santos (7) and Paranagua (8). Cities (3) and (4) will be considered here only as transshipment points.

Figure 2 reformulates the transportation system of Figure 1 as a capacitated network composed of nodes and arcs. A node may represent an origin of flow (producing regions 1 and 2), a transshipment point (3 and 4), or terminal facilities (7 and 8).[3/] An arc is a linkage between two nodes with shipments permitted from one node to another as indicated by arrows.[4/] Each arc is described by its endpoints i and j, and by three parameters (C_{ij}, l_{ij}, U_{ij}): the cost, c_{ij}, of sending a unit of flow between nodes i and j and a lower (lij) and upper uij) bound on the units of flow permitted between two nodes during some specified time interval such as a day, month, or year. Capacities are here defined in ten ton units, and costs in dollars

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[3/] A node may simultaneously represent a terminal and a transshipment point (5 and 6) or an origin and a transshipment point (2).

[4/] Notation in network analysis is not uniform. Conventions adopted in this paper are similar to those used in Potts and Oliver, Taaffe and Gauthier, and King *et al*.

per ten tons. Any number of arcs may connect the same two nodes as long as the parameters for any two are not all identical.

Several arcs in Figure 2 have been assigned cost and capacity parameters by way of illustration.[5/] Node DO is a "dummy" origin which serves as the source of flow for the network. The parameters of the dummy arcs (DO, 1) and (DO, 2) connecting the dummy origin with the "real" origins 1 and 2 indicate that 100 units are available for shipment from producing region 1 and 115 from producing region 2. The zero costs indicate that production costs do not enter into the solution. All production from region 1 can be transported by road to node 6 at $30 per unit, or up to 50 units may be shipped from node 1 to node 2 by road at $20 per unit. From node 2, a maximum of 50 units may move by rail to node 3 at $5 per unit, and an additional 100 units may move by truck for $10 per unit. Note DD is a dummy destination serving as the "sink" for all flows in the network. The lower bounds on the arcs leading to DD are exogenously determined demands. The values of 50 on arc (5, DD) and 30 on (6, DD) indicate that 50 units must be sent to node 5 (Sao Paulo) and 30 units to node 6 (Curitiba). Since the upper bounds are set at the same values, no additional units may flow to these two nodes. Any remaining units which flow through the system will be exported through either of the ports, as given by the arbitrarily large ("L") upper bounds on arcs (7, DD) and (8, DD).

[5/] All arcs are assigned the three parameters in any real problem. The data problems encountered in estimating costs and capacities are the same as those for other methodologies.

Costs on all dummy arcs are set at zero so they do not influence the optimal solution of the real network. The arc (DD, DO) is explained below.

Intermodal Transfer Costs

The simple network of Figure 2 makes no allowance for transfer costs between carriers. This assumption is easily relaxed as shown in the subnetwork of Figure 3. Each node is "split" into two nodes connected by dummy arcs such as (2R, 2) and (2, 2R). The parameters on these arcs indicate that it costs $1 per unit to transfer cargo from truck to rail and $2 from rail to truck. A highway-rail transfer capacity of 30 units exists at location 2, while 50 units can be transferred over all other arcs.

Increasing Costs

An assumption of constant unit costs (C_{ij}) for any origin-destination pair underlies the 3 models discussed earlier. This assumption can be relaxed to allow cases for which C_{ij} increases as volume (X_{ij}) increases. Linear approximations are used to represent the increasing cost situation.

In Figure 4, three arcs link nodes 2 with 3. These all represent the same transportation mode, but different costs. At most 10 units can flow at the lowest cost, $3. An additional 20 units and 50 units can flow at costs of $4 and $5, respectively. Arc (3, 3') establishes a maximum capacity that can flow from 2 to 3, even though the individual arcs represented a rate structure for greater amounts.

Changes in Arcs, Costs and Capacities

The capacitated network model can be easily modified to assess the impact of a) expected increased demand for transportation and storage and b) changing costs and capacities of certain arcs in the system. Such changes are represented simply by changing the respective arc parameters. Changes in relative shipping prices, such as those caused by highway subsidization relative to railways or increases in petroleum prices, are represented in the same fashion. Finally, new facilities are represented by additional arcs. Likewise, the disappearance of facilities such as rail abandonment is represented by the deletion of affected arcs.

Storage and Storage Costs

The preceding discussion was confined to transportation during a single time period. Storage, however, can also be represented in the capacitated network either as a separate system or as a complement to transportation. An illustration of a combined transport-storage network is given in Figure 5. Only one aspect of storage is considered: that involving differential transfer costs.[6/] Such differentials arise when it is necessary to store a commodity to use low cost carriers that become saturated during the harvest season. They may also arise if storage costs vary among locations (say in ports, due to problems of space, congestion, or excessive humidity).

[6/] Storage also occurs due to expected seasonal price increases which are invariant with respect to storage location. The model could be modified to determine the optimum length of storage if the "costs" of price changes were estimated exogenously and assigned to the storage area.

In the transport-storage subnetwork of Figure 5, 100 units are produced in region 1, but only 20 units are demanded at node 8 during the harvest period. Storage from harvest to post-harvest period is presented by flow over arcs (1, 1'), (6, 6'), and (8, 8'). For example, arc (6, 6') represents storage at node 6 for a specified time period. The flows over arcs (1', 6') and (6', 8') are actually over the same physical facilities represented by arcs (1, 6) and (6, 8), but take place during the post-harvest season. The arcs with primed values have greater capacities since the post-harvest season is much longer than the harvest season, giving the transportation facilities more time to move the commodities.

Storage is permitted in the producing region (node 1) at $5 per unit up to 50 units for the post-harvest period, at node 6 ($3 per unit to 50 units), or at the port (node 8) for $9 per unit to 40 units.[7/] This example is solved below.

An Efficient Solution to the Capacitated Network Problem

The Fulkerson "Out-of-Kilter" Algorithm (OKA) is an efficient instrument for solving capacitated network problems even for very large networks [Bradley; Fulkerson; Potts and Oliver;

[7/] These values are hypothetical. Higher port costs are realistic due to greater congestion. The dummy arcs (8', DD) and (DD, DO) could have been assigned upper capacities of 80 and 100, respectively, without changing the solution. An arbitrarily large upper limit (L=1,000), however, would not restrict the solution if supply were greater than 100 units and more than 80 units could be shipped to node 8' after the harvest. This frequently occurs in multiple origin-multiple destination models.

Durbin and Kroenke; Ford and Fulkerson].[8/] The cost and capacity parameters of all arcs (including those representing demands and supplies) must be estimated exogenously. Supply may be equal to or greater than the sum of the amounts demanded. The algorithm determines the maximum set of flows, x_{ij}, so as to minimize the total transfer costs including transport, storage and other costs assigned to the arcs.[9/] That is, the OKA minimizes

(1) $\sum_{i,j} c_{ij}\, x_{ij}$ for all i and j subject to

(2) $l_{ij} \leq x_{ij} \leq u_{ij}$ for all i and j and

(3) $\sum_{j} x_{ji} - \sum_{j} x_{ij} = 0$ for all i

where all symbols are defined as previously.

The last condition is the conservation of flow principle that the total flow into a node must equal the total flow out of it. Thus, in order to solve the problem of Figure 5, a dummy arc (DD, DO) must be added to complete the system, avoiding

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8/ A program generously made available to the authors by Dr. H. L. Gauthier of The Ohio State University is designed to handle up to 1,000 nodes and 3,000 arcs. It is currently being updated to take advantage of modern computer capacities and additional efficiencies [Wollmer]. Faster solution algorithms are becoming available. However, the size of problems analyzed by most agricultural economists does not justify search for the most efficient algorithm. Furthermore, many of the new algorithms are proprietary.

9/ The maximum flow is determined by the minimal cut-set [Potts and Oliver, p. 43]. If all "supply" can be forced through the network, the supply arcs constitute the cut-set (i.e., the maximal flow = available supply). Thus, the maximal flow is given flow and will be allocated to the least cost arcs.

loss of flow at the source (DO) and gain of flow at the sink (DD).[10/]

The OKA determines endogenously the following parameters: 1) the optimal flows (x_{ij}); 2) net arc costs ($\bar{c}_{ij}$ or CBAR values); 3) node prices (Π_i); and 4) kilter numbers.

Node prices (Π_i) are recalculated at each iteration so that increases in commodity flow are along the least expensive paths. They are relative prices and are indicative of locational advantages or rents. They determine, for given arc costs (c_{ij}), the net arc cost:

(4) $\bar{c}_{ij} = c_{ij} - (\Pi_j - \Pi_i)$

Given these parameters, each arc has a kilter state and kilter number as defined below [Fulkerson, pp. 20-21]:

	Kilter State	Kilter Number
(a)	$\bar{c}_{ij} > 0,\ x_{ij} = l_{ij}$	0
(b)	$\bar{c}_{ij} = 0,\ l_{ij} \leqq x_{ij} \leqq u_{ij}$	0
(c)	$\bar{c}_{ij} < 0,\ x_{ij} = u_{ij}$	0
(a_1)	$\bar{c}_{ij} > 0,\ x_{ij} < l_{ij}$	$l_{ij} - x_{ij}$
(b_1)	$\bar{c}_{ij} = 0,\ x_{ij} < l_{ij}$	$l_{ij} - x_{ij}$
(c_1)	$\bar{c}_{ij} < 0,\ x_{ij} < u_{ij}$	$\bar{c}_{ij}(x_{ij} - u_{ij})$
(a_2)	$\bar{c}_{ij} > 0,\ x_{ij} > l_{ij}$	$\bar{c}_{ij}(x_{ij} - l_{ij})$
(b_2)	$\bar{c}_{ij} = 0,\ x_{ij} > u_{ij}$	$x_{ij} - u_{ij}$
(c_2)	$\bar{c}_{ij} < 0,\ x_{ij} > u_{ij}$	$x_{ij} - u_{ij}$

The first three states are "in-kilter" since they satisfy the feasibility criterion (2) and optimality criteria that arcs with negative CBAR values have maximum feasible flows and those with

10/ Recall the addition of this arc in Figure 2.

positive CBAR values have minimum feasible flows. States c_1 and a_2 are feasible but not optimal, while the remaining states are infeasible. Each iteration works to lower the positive kilter number associated with an out-of-kilter arc without increasing the kilter number of any arc. Any changes which occur on any arc will bring it toward one of the optimal states (a), (b) or (c). A single iteration may lower the kilter numbers of several arcs. Thus, all changes which occur are toward optimality for every affected arc and for the network as a whole. In addition, the OKA computational routine may be initiated with any flow (feasible or infeasible) using either the primal or the dual. The optimal solution to a problem furnishes a starting point for post-optimal analysis, permitting rapid solutions to sub-problems where arcs are added or assigned different parameters. These factors make the algorithm extremely efficient and result in lessened demands on computer time and capacity than the traditional linear programming formulations.

An Illustration: OKA Solution and Interpretation

The optimal OKA solution to the problem of Figure 5 is given in Table 1. The x_{ij} values are the flows constituting the least cost means of forcing the given supply through the network (e.g., 30 units are sent from node 1 to node 6 in the post-harvest period as given by $x_{1'6'} = 30$ on arc (1', 6').

The node prices Π_i reveal the location rents of the nodes with respect to the destination (node 8). They are relative prices (for example, the price at node 6 is \$20 more than at node 1).

CBAR values are similar in interpretation to dual variables in linear programming. In this example they indicate the change in cost associated with a one unit change in capacity in some arc of the network. Negative CBAR values imply that the flow over the arc is at its maximum value and that savings could be obtained if the capacity of the arc were expanded and flows diverted to it from more costly paths. Thus, arcs with the largest negative CBAR values constitute major bottlenecks to a more efficient transfer of goods and are useful instruments for post-optimal (sensitivity) analysis. The only bottleneck in the system of Figure 5 is arc (6, 6'), that is, storage at node 6. If that capacity were increased, flow could be rerouted to meet the demands with a cost reduction of $2 per unit until a bottleneck developed on another arc. Positive CBAR values imply flow is at the minimum value. These values represent the cost to the system of increasing flow over the associated arcs by one unit. If the arcs indicate demand requirements, this is the total cost of sending one additional unit of flow through the system if supply were available. For arc (8, DD), this cost is $25.

All kilter numbers in the table are zero, indicating that the solution is optimal. A positive kilter number would indicate an arc with a non-optimal or infeasible flow. At least one arc is "out-of-kilter" until the optimal solution is obtained (hence the name of the algorithm). If one or more arcs cannot be brought into kilter, the problem is infeasible.

Extensions

The Fulkerson algorithm provides a highly efficient solution to the simple transportation, transshipment with one intermediate point, the shortest path and maximal flow problems as special cases. Multiple commodities using the same network present a modelling problem for all types of models. The assumption of product homogeneity may be relaxed if two commodities may be expressed in common units and treated as an aggregate homogeneous commodity (e.g., 5 tons corn = 4 tons soybeans = 4 units of flow). Alternatively, researchers using CN methods have assumed that all of one product will be allocated before any of the subsequent product will be allocated [Hu; Jewell]. The algorithm can be converted from a static instrument into a tool for constructuring maximal dynamic flows [Ford and Fulkerson, 1957]. More work is required to adapt the methodology to the multicommodity case.

Additional extension of CN includes Wollmer's reformulation to allow for node capacities as well as arc capacities. Panagiotakopoulos has presented a network model which permits flow transformation and positive gains along the arcs.

Summary and Conclusions

Transportation models developed to date have been most effective in studying problems of developed economies. Transport-storage networks in LDC's, however, frequently require the analysis of multiple transshipment points and capacity constraints which are subject to rapid alteration due to massive investment programs. The capacitated network approach outlined in this paper offers the researcher an effective tool for studying alternative investments in transportation and storage systems when the improvements may significantly affect the entire system.

Empirical applications have lagged far behind the theoretical developments of the capacitated network approach. This paper has attempted to bridge that gap by illustrating an important class of commodity storage and transport problems for which the capacitated network model is the appropriate analytical instrument. The applications of the network model referenced in this paper provide information on how agriculturalists have recently used this instrument to study grain transportation problems in Ohio and Brazil.

Figure 1: Simplified Transportation System for Northwestern Sao Paulo State and State of Parana, Brazil

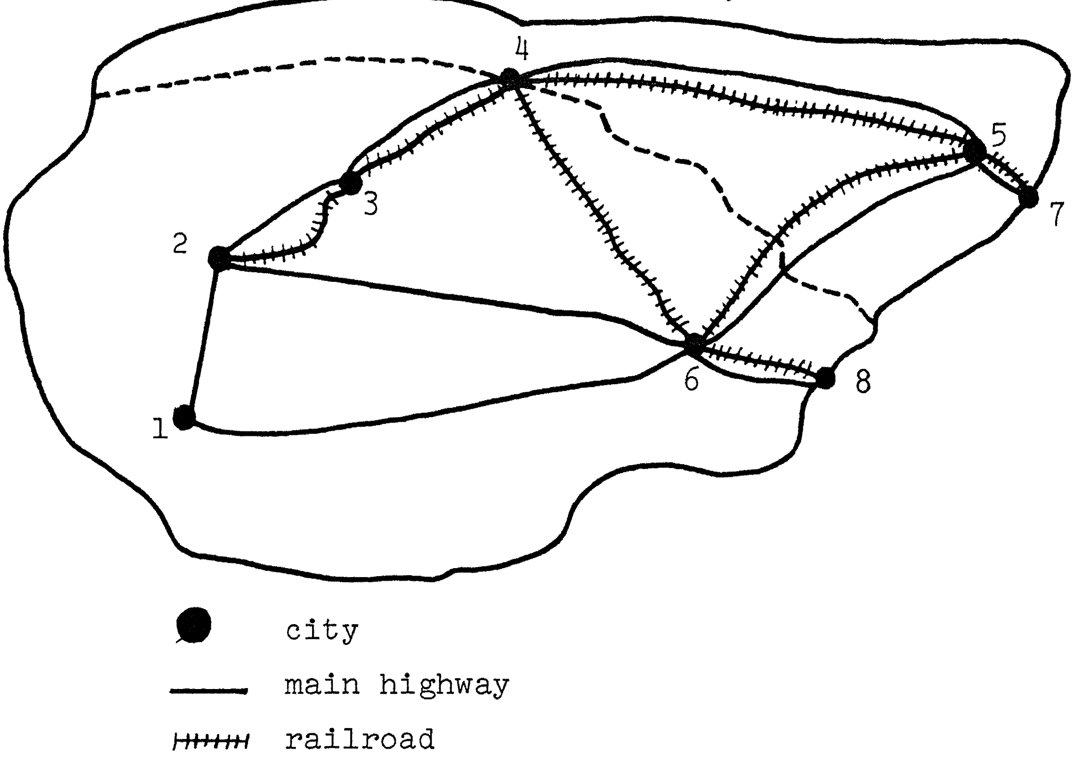

Figure 2: The Transportation System of Figure 1 Depicted as a Capacitated Network

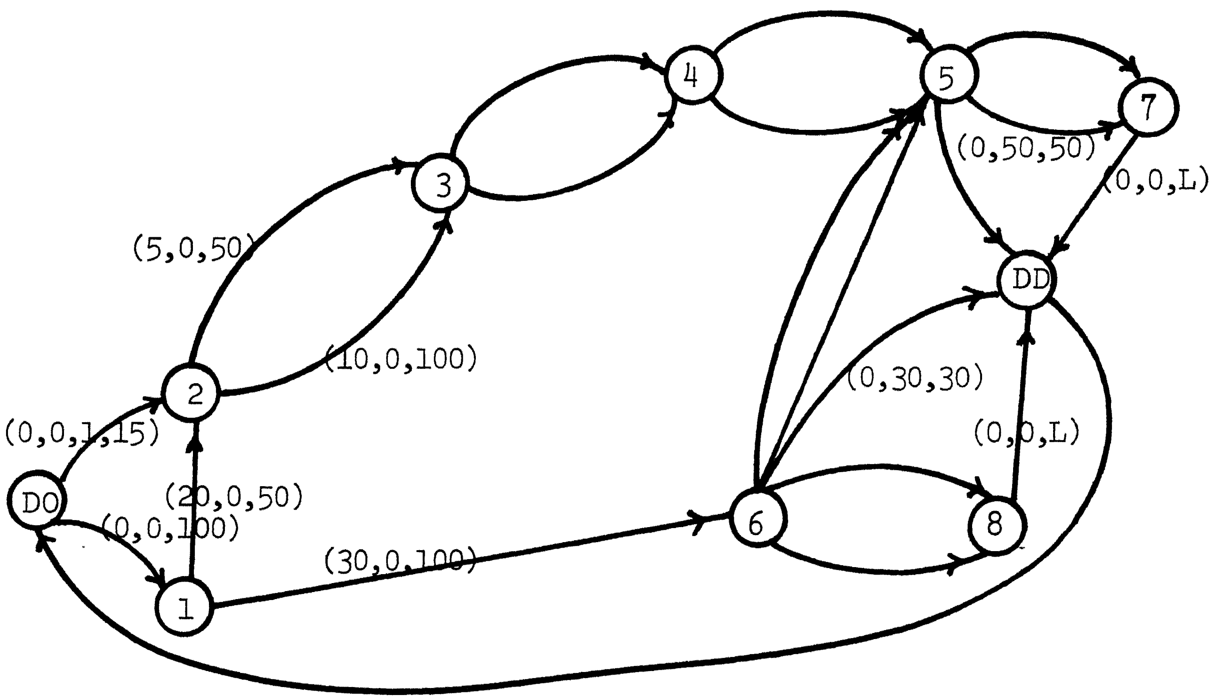

Figure 3: Intermodal Transfer Costs

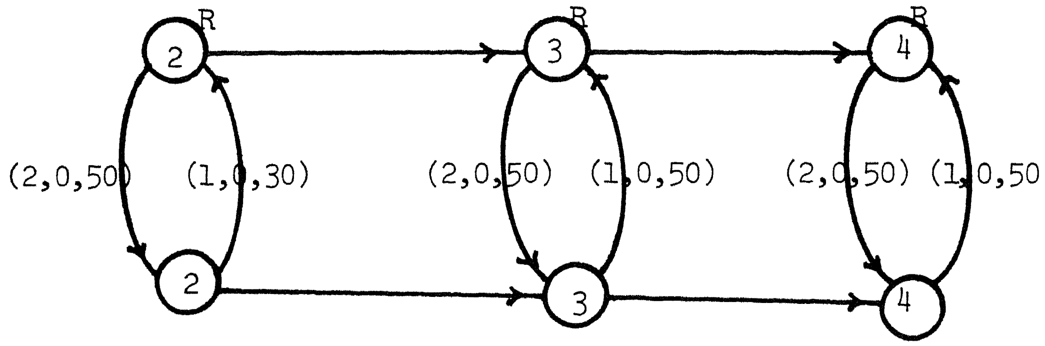

Figure 4: Increasing Costs

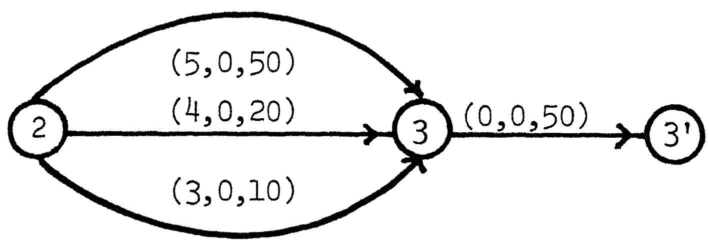

Figure 5: Storage and Storage Costs

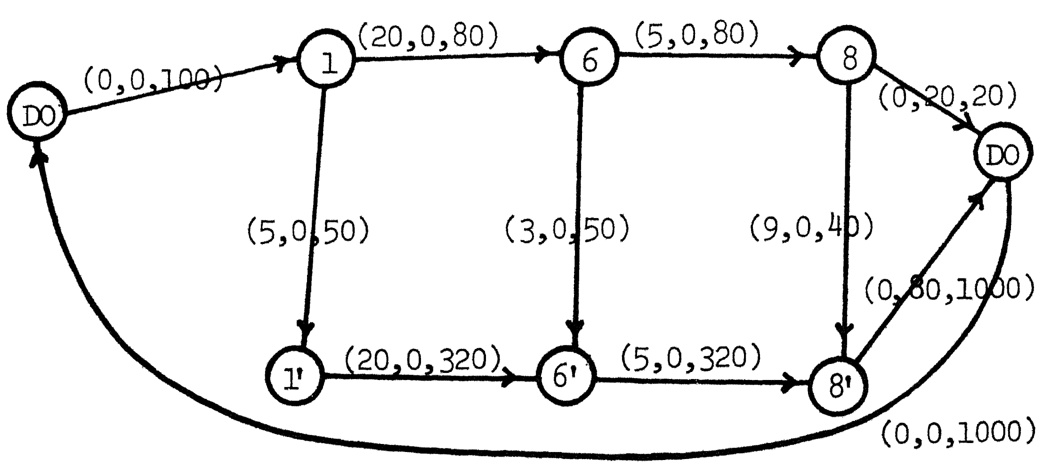

Table 1. Optimal OKA Solution
For Transport-Storage Problem of Figure 5*

Arcs i	j	Cost Per Unit	Lower Limit (Units)	Upper Limit (Units)	x_{ij} Optimal Flows	Net Arc Cost (CBAR)	Kilter N	Total Transport Costs on Arc (C_{ij} times X_{ij})
D	1	0	0	100	100	0	0	0
1	1'	5	0	50	30	0	0	150
1	6	20	0	80	70	0	0	1400
1'	6'	20	0	320	30	0	0	600
6	6'	3	0	50	50	-2	0	150
6	8	5	0	80	20	0	0	100
6'	8'	5	0	320	80	0	0	400
8	8'	9	0	40	0	4	0	0
8	DD	0	20	20	20	25	0	0
8'	DD	0	80	1000	80	30	0	0
DD	DO	0	0	1000	100	0	0	0

Total Transfer Cost = $2,800

* Node prices (Π_i) are $0 for nodes DO, 1, and DD; $5 for node 1'; $20 for 6; $25 for 6' and 8; and $30 for 8'.

REFERENCES

Bradley, Gordon H., "Survey of Deterministic Networks," AIIE Transactions, No. 7, (1975) pp. 222-234.

Casetti, Emilio, "Optimal Location of Steel Mills Serving the Quebeck and Southern Ontario Steel Market," The Canadian Geographer, Vol. 10, No. 1, (1966) pp. 27-39.

Dantzig, George B., Linear Programming and Extensions, Princeton: Princeton University Press, 1963, pp. 335-403.

Durbin, E. P. and Kroenke, D. M., "The Out-of-Kilter Algorithm: A Primer," Rand Memorandum RM-5472-PR, Santa Monica, December 1967.

Fedeler, Jerry A., Earl O. Heady and Won W. Koo, "A National Grain Transportation Model," in Heady, Earl O., and Uma K. Srivastava (eds.) Spatial Sector Programming Models in Agriculture, Ames: Iowa State University Press, 1975, pp. 452-479.

Feldens, Aray M., "A Transportation-Storage Network Analysis of Wheat and Soybeans in Rio Grande do Sul, Brazil," Unpublished Ph.D. dissertation, Department of Agricultural Economics and Rural Sociology, The Ohio State University, 1978.

Ford, L., and D. Fulkerson, Flows in Networks, Princeton: Princeton University Press, 1962.

Fulkerson, D. R., "An Out-of-Kilter Method for Minimal-Cost Flow Problems," J. Soc. Indust. Appl. Math., Vol. 9, No. 1, (March, 1961), pp. 18-27.

Gauthier, H. L., "Least Cost Flows in a Capacitated Network: A Brazilian Example," E. Hurst (ed.), Transportation Geography, New York: McGraw-Hill, 1973, pp. 172-186.

Goldman, Thomas A., "Efficient Transportation and Industrial Location," Papers and Proceedings of the Regional Science Association, Vol. 4, (1958) pp. 91-106.

Hu, T. C., Integer Programming and Network Flows, Addison-Wesley Publishing Co., Inc., Reading Mass.: 1969.

Jewell, W., "Multi-Commodity Network Solutions," RAND, AD-639-672, September 1966.

Kane, Michael D., "Impact of Rail Reorganization on Grain Transfer Activities, Flow Patterns, and Costs in Central and Southwestern Ohio," Unpublished M.S. thesis, Department of Agricultural Economics and Rural Sociology, The Ohio State University, 1978.

King, Gordon A., and Samuel H. Logan, "Optimum Location, Number and Size of Processing Plants with Raw Product and Final Product Shipments," J. of Farm Econ., Vol. 46, No. 1 (February, 1964), pp. 94-116.

King, L., et al., "Optimal Transportation Patterns of Coal in the Great Lakes Region," Economic Geography, Vol. 47, (1971), pp. 401-413.

King, Richard A. (ed.), Interregional Competition: Research Methods, Raleigh: North Carolina State Print Shop, 1963.

King, Richard A. and William R. Henry, "Transportation Models in Studies of Interregional Competition," J. of Farm Econ. Vol. 41, No. 5, (December 1959), pp. 997-1011.

Kriebel, Charles H., "Warehousing with Transshipment Under Seasonal Demand," Journal of Regional Science, Vol. 3 (1961), pp. 57-69.

Ladd, George W. and Dennis R. Lifferth, "An Analysis of Alternative Grain Distribution Systems," Amer. J. Agr. Econ. 57, (1975), pp. 420-430.

Leath, Mack N. and Leo V. Blakley, "An Interregional Analysis of the U.S. Grain Marketing Industry, 1966-67," USDA/ERS/ Oklahoma State University, Technical Bulletin No. 1444, November 1971.

McCurdy, S. J., et al., "A Capacity and Cost Study of the South Island Transport Network in Relation to Container Movements," Department of Geography, University of Canterbury (xerox).

Morrill, Richard L., and William L. Garrison, "Projections of Interregional Trade in Wheat and Flour," Economic Geography, Vol. 36, No. 2 (1960), pp. 116-126.

Muraco, William A., "Intraurban Accessibility," Economic Geography, Vol. 48, No. 4 (October 1972), pp. 388-405.

Panagiotakopoulos, D., "Multi-Commodity Multi-Transformed Network Flows with an Application to Residuals Management," Management Science, Vol. 22, No. 8 (April 1976), pp. 874-882.

Potts, Renfrey B., and Robert M. Oliver, Flows in Transportation Networks, New York: Academic Press, 1972.

Rhody, Donald D., "An Illustration of the Method of Reduced Matrices for the Solution of a Generalized Distribution Problem," in Richard A. King (ed.), Interregional Competition: Research Methods, Raleigh: North Carolina State Print Shop (1963), pp. 79-92.

Sinclair, J. G. and C. C. Kissling, "A Network Analysis Approach to Fruit Distribution Planning," Proceedings of the 6th New Zealand Geography Conference, Christchurch (1971) pp. 131-136.

Stevens, Benjamin H., "Linear Programming and Location Rent," Journal of Regional Science, Vol. 3 (Winter 1961) pp. 15-25.

Taaffe, Edward J., and H. L. Gauthier, Geography of Transportation, Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1973.

Takayama, T., and G. G. Judge, "Spatial Equilibrium and Quadratic Programming," J. of Farm Econ., Vol. 46, No. 1, (February, 1964), pp. 67-93.

Von Boventer, Edwin, "The Relationship Between Transportation Costs and Location Rent in Transportation Problems," Journal of Regional Science, Vol. 3, No. 2, 1961, pp. 27-40.

Walker, F. E., "Transportation and Spatial Equilibrium Models for Interregional Analysis," in Workshop in Interregional Competition, Oklahoma State University, Department of Agricultural Economics, 1966.

Wollmer, R., "Maximizing Flow Through a Network with Node and Arc Constraints," Santa Monica: RAND, RM-5383-PR, July 1967.

Wright, Bruce H., "Regional and Sectoral Analysis of the Wheat-Flour Economy: A Transportation Study," USDA/ERA, Marketing Research Report No. 858, October 1969.

Wright, Charles L., "The Economics of Grain Transportation and Storage: A Brazilian Case Study," Unpublished Ph.D. dissertation, Department of Agricultural Economics and Rural Sociology, The Ohio State University, 1977.